

HSRC RESEARCH OUTPUTS

4074

Understanding learner performance in Number Concepts

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Main questions

- How did learners perform
- Common problems experienced
- Trends between 1999 and 2003



Overview of framework and organization of 2003 TIMSS Mathematics test

- Large number of questions included to cover curriculum.
- Example questions released for public use.
- Tests organized along two domains:
 - Content domain
 - Cognitive domain



Mathematics content domains

Whole numbers	Targeted time	Number of MCQ	Number of CRC
Numbers	30%	43	14
Algebra	25%	29	18
Measurement	15%	19	12
Geometry	15%	22	9
Data	15%	15	13



Mathematics cognitive domains

Cognitive Domain	Targeted time
Knowing Facts and Procedures	15%
Using Concepts	20%
Solving Routine Problems	40%
Reasoning	25%



Methodology

- Data collected on performance of Grade 8 learners in 1999 and 2003.
- TIMSS uses a sophisticated process of scaling to ensure comparability among participating countries and over time periods.
- For this study results were reported on average percentage of correct responses.



Methodology (continue)

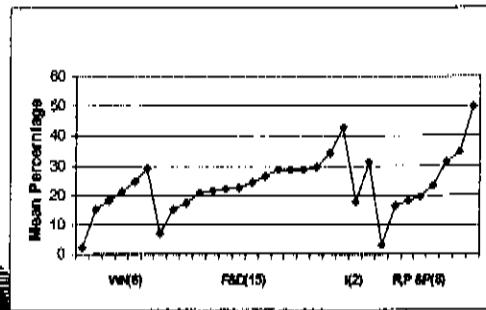
□ Categories investigated

- Content domain
- Cognitive domain
- Question type
- Gender
- School type

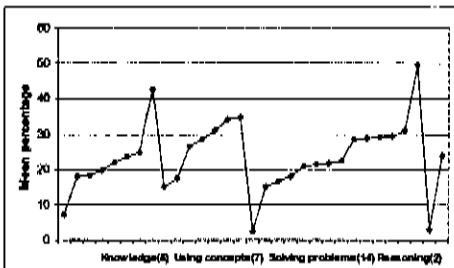
□ Performances on question included in both 1999 and 2003 were compared.



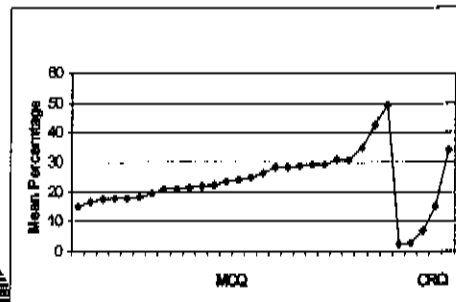
Performance for released questions per topic under the Number Content Domain



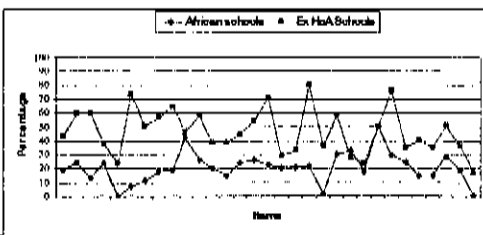
Performance for released questions per cognitive domain



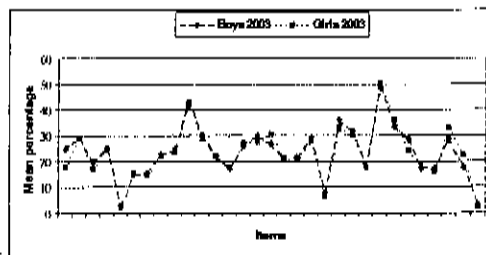
Performance per question type

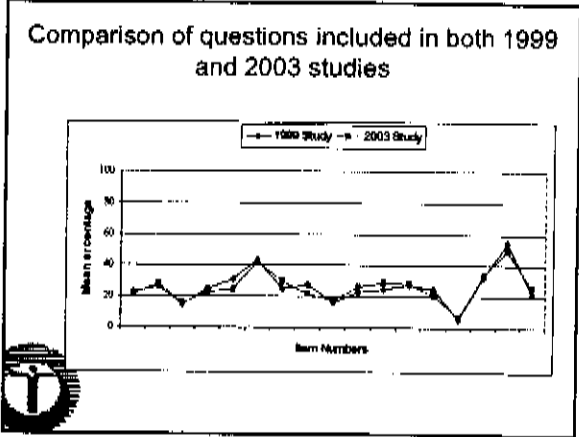


Performance per School type (ex-Department)



Performance per gender





Example 1: Whole numbers

$\square \square \square \square$
 The four digit digits are to be arranged from largest to smallest to form a two-digit number. The same four digits are then to be arranged from smallest to largest to form another two-digit number. What is the difference between the two resulting two-digit numbers?

Ⓐ 3726
 Ⓑ 4726
 Ⓒ 5426
 Ⓓ 8142
 Ⓔ 8122

Content Domain:
 Number

Main Topic:
 Whole numbers

Cognitive Domain:
 Solving Realistic Problems

Key:
 B

Example 2: Whole numbers

Which of these is closest to $11^2 + 12^2$?

Ⓐ 20 + 20
 Ⓑ 31 + 41
 Ⓒ 120 + 20
 Ⓓ 120 + 30

Content Domain:
 Number

Main Topic:
 Whole numbers

Cognitive Domain:
 Estimating Facts and Procedures

Key:
 D

Example 3: Fractions & Decimals

In the figure, how many MORE small squares need to be shaded so that $\frac{4}{6}$ of the small squares are shaded?

Ⓐ 6
 Ⓑ 4
 Ⓒ 2
 Ⓓ 1

Content Domain:
 Number

Main Topic:
 Fractions and Decimals

Cognitive Domain:
 Using Concepts

Key:
 B

Example 4: Fractions & Decimals

In a group of children, 15 have birthdays during the first half of the year, and 14 have birthdays during the second half of the year. What fraction of the group have birthdays during the first half of the year?

Ⓐ $\frac{14}{29}$
 Ⓑ $\frac{11}{29}$
 Ⓒ $\frac{16}{24}$
 Ⓓ $\frac{15}{26}$
 Ⓔ $\frac{29}{16}$

Content Domain:
 Number

Main Topic:
 Fractions and decimals

Cognitive Domain:
 Solving Realistic Problems

Key:
 A

Example 5: Integers

What is the value of $4 - 5 + (-2)$?

Ⓐ 11
 Ⓑ 8
 Ⓒ -8
 Ⓓ -9

Content Domain:
 Number

Main Topic:
 Integers

Cognitive Domain:
 Knowing Facts and Procedures

Key:
 C

Example 6: Ratio & Percent

In a group of children, 16 have birthdays during the first half of the year and 14 have birthdays during the second half of the year. What fraction of the group have birthdays during the first half of the year?

A $\frac{14}{30}$

B $\frac{14}{16}$

C $\frac{16}{14}$

D $\frac{16}{30}$

E $\frac{30}{16}$

Content Domain

Number

Key Topic

Fractions and Decimals

Cognitive Domain

Solving Real-World Problems

Key

0

Example 7: Ratio & Percent

A computer club had 40 members, and 60% of the members were girls. Later, 30 boys joined the club. What percent of the members now are girls? Show the calculations that led to your answer.

Answer: _____

Content Domain

Number

Key Topic

Fractions, Decimals, and Percents

Cognitive Domain

Reasoning and Problem Solving

Key

See scoring guide

Findings

- Average ranged between 18% and 35%
- Whole numbers most difficult topic.
- Performed poorly at all cognitive skills.
- Found MCQ easier than CRQ.
- No significant gender difference.
- Ex-African schools performed well below ex-Model C schools.
- Difference between 1999 and 2003 relatively small.

Findings (continue)

- Analysis of individual items revealed that many learners
 - Experienced difficulties with operations using numbers
 - Confusion with place value
 - Did not understand common fractions
 - Did not understand the concept of percentages
 - Showed a lack of knowledge of exponents
 - Experienced difficulties with questions that required the use of knowledge of ratios and proportions (3 questions not in curriculum).

Conclusion

- Shortcomings in learners knowledge and skills in the number domain (LO1).
- Affects performance in other domains (LOs).
- Timely support to improve knowledge and skills in number domain could improve overall performance.

Thank you!

Understanding Learner Performance in Number Concepts

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Introduction

This presentation focuses on the achievement of South African learners in the Trends in International Mathematics and Science Study (TIMSS 2003) in the area of number concepts. Responses from learners (actual correct and incorrect answers) were analysed in greater detail in an attempt to reveal what learners know about numbers, number operations and the use of number operations in solving problems..

The main questions in this study were:

- How did South African Grade 8 learners perform in the area of numbers?
- What were the common problems experienced in number concept questions that may have contributed to poor performance?
- Was there any difference in learners' performance on questions that were included in both the 1999 and 2003 studies?

Before the discussion of the performance on the released number items an overview of the framework and organization of the TIMSS mathematics test in 2003 are provided.

The TIMSS 2003 mathematics test was organized along two domains, a content domain and a cognitive domain. The content domain was divided into five sections, while the cognitive dimension had four domains. The two tables below provide an overview of the target percentage of testing time across the content and cognitive domains.

Table 1. Mathematical content domains

Content domain	Target % of testing time	Number of items in TIMSS 2003 test	
		Multiple-choice	Constructed response
Numbers	30%	43	14
Algebra	25%	29	18
Measurement	15%	19	12
Geometry	15%	22	9
Data	15%	15	13
Total	100%	128	66

This table indicates that the greatest emphasis in the mathematics instruments was on the Number Content Domain. Most testing time and most questions were devoted to this domain.

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Performance on Whole Numbers

South African learners performed very poorly on all the released questions under whole numbers. None of the questions were answered correctly by more than one third of the learners. An investigation of the incorrect options selected or incorrect answers produced per question by learners reveals that mistakes made in algorithmic procedure and computations contributed most to learners' errors. Confusion with place value probably contributed to incorrect answers being selected by the majority of learners in two of the questions. Lack of knowledge of exponents contributed to the majority of learners answering another question incorrectly.

The question which produced the lowest scores under this topic, with only 2.3% correct responses, was a constructed response question which required learners to calculate the number of packs of paper needed by teachers to publish six magazines per year. Learners had to solve the problem by applying knowledge of computations and algorithmic procedures.

The lowest scoring multiple-choice question required learners to select the answer closest to $11^2 + 9^2$ from four possible answers. The question was answered correctly by 18.3% of the learners. More than 47% learners selected one incorrect option. These learners arrived at the answer by multiplying the approximate value of 11 and 9 by two, i.e. $10 \times 2 + 10 \times 2$ to get the answer $20 + 20$. This indicates that learners have not grasped the concept of exponents.

Performance on Fractions and Decimals

More than 40% of learners were able to answer the easiest question on this topic correctly. Only 6.8% of learners answered the most difficult question correctly. An investigation into the frequent errors made in questions on fractions and decimals revealed that mistakes made with algorithmic procedure and computations contributed to many of these errors. The most frequent incorrect answers selected for three of the questions indicated that learners experience problems with operations involving common fractions. Difficulties with operations with decimal fractions probably contributed to the selection of the most frequent incorrect answers for another three of the questions. Lack of understanding of place value contributed to incorrect answers being selected in four questions.

Performance in Integers

The percentage of learners who answered the two questions relating to integers correctly was 17.9% and 30.9% respectively. An analysis of answers to the two questions showed that learners experienced difficulties with the concept of multiplying with negative numbers in both questions and that they were uncertain about the order in which operations should be carried out in one of the questions.

Performance in Ratio, Proportion and Percent

The question which produced the highest scores of all the released questions on Numbers was from the topic *ratio, proportion and percent*. Slightly fewer than 50% of learners answered the question correctly. In this question learners were asked to determine the number of laps that Alice ran in equivalent ratios. This is a multi-step word problem making use of operations with common fractions. The high score for this question runs counter to the general trend illustrated earlier in this document, that learners found operations with common fractions difficult. The unexpectedly high percentage of 51% of learners from ex-African schools who were able to select the correct answer was one of the factors that contributed to the overall high score for this question.

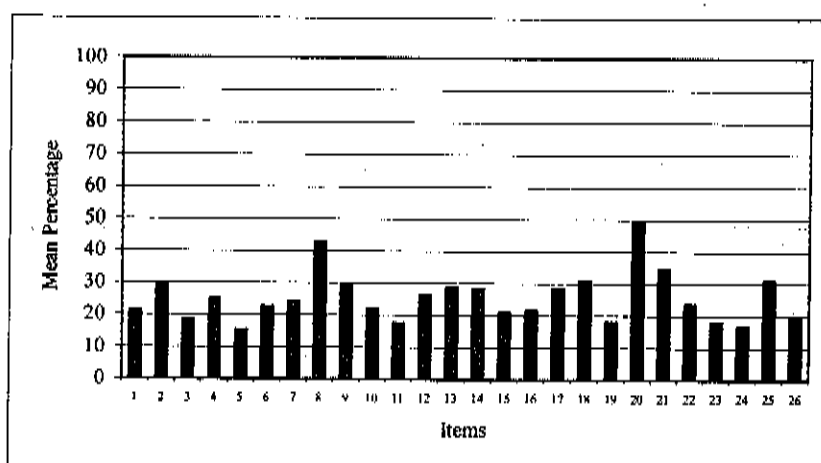
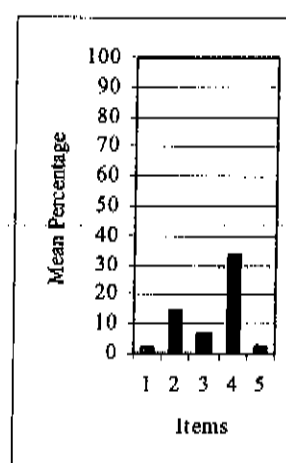
The lowest scoring question under this topic was a constructed response question, resulting in only about 2% correct or partially correct answers. This question requires the highest order of cognitive skills, i.e. reasoning. Learners had to (a) use ratio and percentages and (b) use computations with

Table 4. Distribution of questions and mean percentages per question type

Topic	Number of MCQ	Number of CRQ	Mean % for MCQ	Mean % for CRQ
Whole Numbers	4	2	23.4	8.7
Fractions and Decimals	13	2	25.2	20.6
Integers	2	-	24.4	-
Ratio, Proportion and Percent	7	1	27.5	2.9
Total	26	5	25.5	12.3

An investigation of the learners' performance per question type, as illustrated in Table 4, shows that learners perform better in the multiple-choice questions than in the constructed response questions. This is a trend that is supported by many studies on student performance in Mathematics.

Figure 1 illustrates the performance of learners on the multiple-choice questions and Figure 2 illustrates the performance of learners on the constructed response questions.

**Figure 1.** Performance on MCQ**Figure 2.** Performance on CRQ

The average performance on most of the multiple-choice questions ranged between 15% and 30%, while the average performance for most of the constructed response questions were below 15%. According to Figure 2, learners performed relatively better in one of the constructed response questions. The question assessed common fractions and required learners to use concepts of common fractions to write a fraction that was smaller than a given common fraction. The question did not require learners to show any steps or to give any explanations. The difficulty that learners usually experience with CRQ, i.e. expressing themselves in words, was thus not applicable to this question.

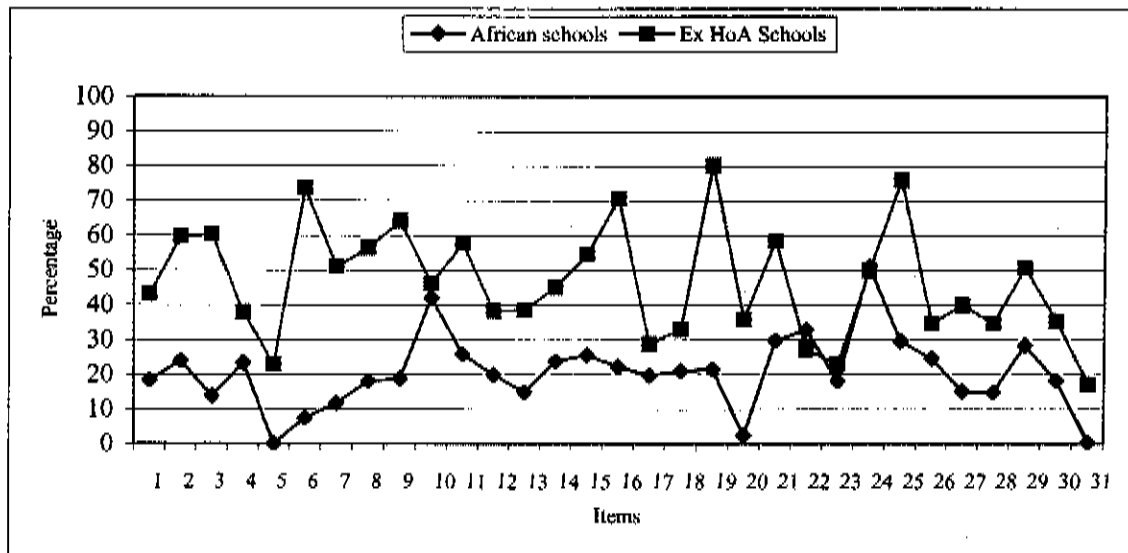


Figure 4. Performance of learners per School Type

In most of the questions, learners from the African Schools performed well below those who were classified under the ex-HoA schools. There was a difference of 25% between the two groups. This could be expected because (a) ex-HoA schools are usually better resourced than ex-African schools, (b) their teachers are usually better qualified and (c) the number of learners per class is usually smaller.

Comparison of performance in 1999 and 2003

Figure 5 illustrates the pattern of performance on the questions that were included in both 1999 and 2003.

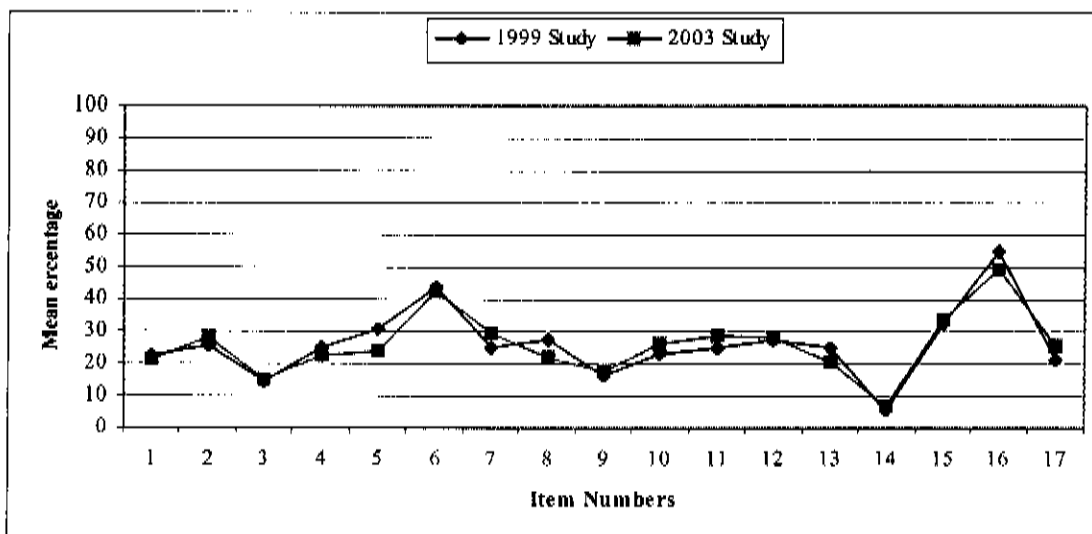


Figure 5: Comparison of questions included in both 1999 and 2003 Studies

The differences between the performances of learners for the two years are relatively small. According to this, there was not much improvement in learners' knowledge and skills in number concepts between 1999 and 2003.

shortcomings in the teaching of numbers. It is generally accepted that a strong content base in numbers and number concepts is fundamental to critical thinking and problem solving and that this forms a basis for performance in the other Learning Outcomes in mathematics.

It is therefore important that learners are supported in improving their knowledge and skills in the number domain. Timely support could enable them to improve performance in the other mathematics domains.

References

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